**Assignment – Derivatives Securities 2022**

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**Data assigned:** *November - December 2019*

**3.2. Estimation**

(1) The standard deviation of the daily changes in the jet fuel price index is **1,55%**

(2) The standard deviation of the daily changes in the heating oil futures price index is **1,35%**

(3) The coefficient of linear correlation between the daily changes in the two prices is **0,89168**

Daily changes were calculated the following way:

**3.3 Hedging**

Following the minimum-standard deviation hedge formula:

The value of the hedge position h is equal to -1,02612173, meaning that we would take a short position in 1,02612173 units in heating oil futures contracts.

For the standard deviation of the minimum standard deviation portfolio, we have:

The minimized standard deviation of the hedging portfolio is 0,70%.

**3.4 A prediction exercise**

Since we are dealing with daily data, but want to predict future prices in 10 days, we need to transform the scaling of the used data.

This is done by multiplying the standard deviations with .

From this follows:

For the hedged portfolio

For the unhedged portfolio

is calculated the following way:

As we know from the exercise the drift is equal to 0, which leaves us with the following formula for the log returns:

From this follows that:

Where Z is calculated in excel as =NORM.S.INV(RAND())

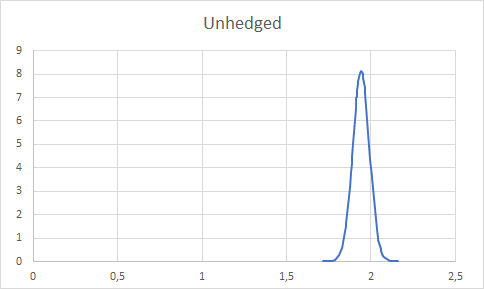
Where T is

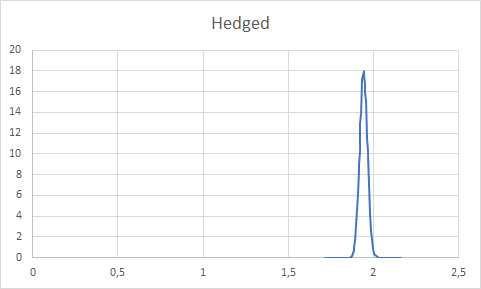
A total of 1000 simulations have been conducted for each portfolio.

The simulated values of the price in 10 days were used in the estimation of the graphs.

As we are hedging against volatility changes, and not against changes in the value of the underlying, the same last price was assumed in both, the hedged and unhedged portfolio.

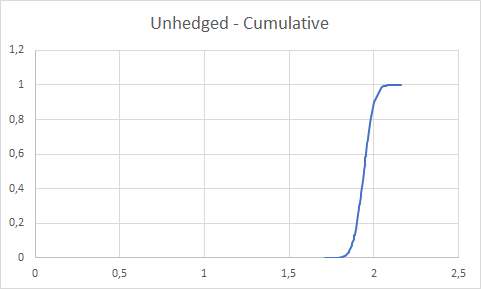
1. Plots of density functions of the unhedged and hedged cashflows

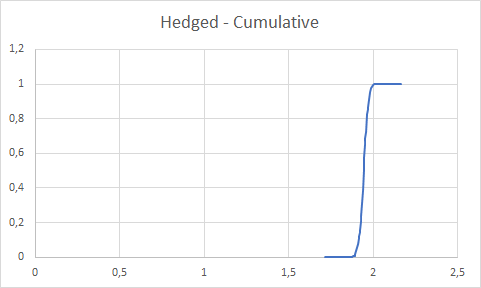




As can be seen from the graphs, the graph for the hedged portfolio is much “slimmer”. This is due to the reduction of volatility in the cash flow.

1. Plots of the cumulative distributions of the unhedged and hedged cashflows





As can be seen from the graphs, the cumulative curve of the hedged portfolio starts to increase significantly later than the one of the graph for the unhedged portfolio. However, the 100% is reached faster in the case of the hedged portfolio. This can be attributed to a more “concentrated” distribution, meaning that much more observations lie closer to the mean than it is the case for the unhedged portfolio.

1. Value of the cashflow c such that the probability of a cashflow greater than c is equal to 5%

Hedged: 1,97857567

* =NORM.INV(0.95,Cash-Flow,)

Unhedged: 2,02280091

* =NORM.INV(0.95,Cash-Flow,)

**Appendix**

**Formulas used in Excel:**

Future prices:

1. Inverse Normal distribution (random number): =NORM.S.INV(RAND())
2. Log Return: =Drift+(1)\*Volatility
3. Price in 10 days: =Initial Value \* exp(2)

Normal distribution graph:

1. =NORM.DIST(Estimated value in 10 days, Expected value in 10 days, Volatility, FALSE)

Cumulative distribution graph:

1. =NORM.DIST(Estimated value in 10 days, Expected value in 10 days, Volatility, TRUE)

Descriptive Statistics:

1. VAR: =VAR.S(Daily Changes)
2. Standard Deviation: =SQRT((1))
3. Correlation: =CORREL(Daily Kerosine Changes ; Daily Heating Oil Future Changes)

**Values of the Time Series:**

|  |  |  |
| --- | --- | --- |
| **Date** | **Kerosine** | **Heating Oil Futures** |
| Oct 31,2019 | 1.80 | 1.88 |
| Nov 01, 2019 | 1.86 | 1.93 |
| Nov 04, 2019 | 1.83 | 1.94 |
| Nov 05, 2019 | 1.87 | 1.96 |
| Nov 06, 2019 | 1.83 | 1.93 |
| Nov 07, 2019 | 1.82 | 1.92 |
| Nov 08, 2019 | 1.83 | 1.92 |
| Nov 12, 2019 | 1.80 | 1.90 |
| Nov 13, 2019 | 1.82 | 1.91 |
| Nov 14, 2019 | 1.80 | 1.91 |
| Nov 15, 2019 | 1.83 | 1.94 |
| Nov 18, 2019 | 1.77 | 1.90 |
| Nov 19, 2019 | 1.73 | 1.86 |
| Nov 20, 2019 | 1.77 | 1.89 |
| Nov 21, 2019 | 1.83 | 1.94 |
| Nov 22, 2019 | 1.84 | 1.93 |
| Nov 25, 2019 | 1.85 | 1.94 |
| Nov 26, 2019 | 1.86 | 1.96 |
| Nov 27, 2019 | 1.85 | 1.95 |
| Dec 02, 2019 | 1.80 | 1.89 |
| Dec 03, 2019 | 1.80 | 1.88 |
| Dec 04, 2019 | 1.83 | 1.92 |
| Dec 05, 2019 | 1.83 | 1.93 |
| Dec 06, 2019 | 1.84 | 1.95 |
| Dec 09, 2019 | 1.83 | 1.94 |
| Dec 10, 2019 | 1.86 | 1.97 |
| Dec 11, 2019 | 1.82 | 1.93 |
| Dec 12, 2019 | 1.84 | 1.95 |
| Dec 13, 2019 | 1.87 | 1.99 |
| Dec 16, 2019 | 1.89 | 2.00 |
| Dec 17, 2019 | 1.92 | 2.03 |
| Dec 18, 2019 | 1.91 | 2.02 |
| Dec 19, 2019 | 1.94 | 2.00 |
| Dec 20, 2019 | 1.93 | 2.00 |
| Dec 23, 2019 | 1.95 | 2.02 |
| Dec 24, 2019 | 1.96 | 2.04 |
| Dec 26, 2019 | 1.98 | 2.05 |
| Dec 27, 2019 | 1.98 | 2.05 |
| Dec 30, 2019 | 1.97 | 2.04 |
| Dec 31, 2019 | 1.94 | 2.02 |
| Jan 02, 2020 | 1.93 | 2.02 |

**Daily Changes in the time series:**

|  |  |  |
| --- | --- | --- |
| **Date** | **Kerosine** | **Heating Oil Futures** |
| Oct 31, 2019 |  |  |
| Nov 01, 2019 | 0.031 | 0.030 |
| Nov 04, 2019 | -0.015 | 0.004 |
| Nov 05, 2019 | 0.020 | 0.008 |
| Nov 06, 2019 | -0.018 | -0.015 |
| Nov 07, 2019 | -0.007 | -0.004 |
| Nov 08, 2019 | 0.003 | -0.001 |
| Nov 12, 2019 | -0.014 | -0.011 |
| Nov 13, 2019 | 0.013 | 0.006 |
| Nov 14, 2019 | -0.013 | 0.003 |
| Nov 15, 2019 | 0.016 | 0.015 |
| Nov 18, 2019 | -0.033 | -0.022 |
| Nov 19, 2019 | -0.026 | -0.023 |
| Nov 20, 2019 | 0.028 | 0.019 |
| Nov 21, 2019 | 0.030 | 0.027 |
| Nov 22, 2019 | 0.008 | -0.007 |
| Nov 25, 2019 | 0.004 | 0.007 |
| Nov 26, 2019 | 0.004 | 0.008 |
| Nov 27, 2019 | -0.004 | -0.006 |
| Dec 02, 2019 | -0.030 | -0.032 |
| Dec 03, 2019 | 0.002 | -0.003 |
| Dec 04, 2019 | 0.019 | 0.023 |
| Dec 05, 2019 | -0.001 | 0.005 |
| Dec 06, 2019 | 0.005 | 0.010 |
| Dec 09, 2019 | -0.007 | -0.004 |
| Dec 10, 2019 | 0.015 | 0.011 |
| Dec 11, 2019 | -0.019 | -0.019 |
| Dec 12, 2019 | 0.009 | 0.011 |
| Dec 13, 2019 | 0.014 | 0.018 |
| Dec 16, 2019 | 0.012 | 0.008 |
| Dec 17, 2019 | 0.018 | 0.014 |
| Dec 18, 2019 | -0.005 | -0.006 |
| Dec 19, 2019 | 0.012 | -0.008 |
| Dec 20, 2019 | -0.003 | -0.004 |
| Dec 23, 2019 | 0.008 | 0.013 |
| Dec 24, 2019 | 0.008 | 0.007 |
| Dec 26, 2019 | 0.008 | 0.008 |
| Dec 27, 2019 | 0.002 | -0.001 |
| Dec 30, 2019 | -0.008 | -0.004 |
| Dec 31, 2019 | -0.012 | -0.010 |
| Jan 02, 2020 | -0.004 | 0.001 |

**Future price simulation:**

A sample of 100 price simulations for the hedged and unhedged portfolio were used, where:

* # indicates the number of simulation
* N(0.1) indicates the inverse normal distribution =NORM.S.INV(RAND())
* Log return indicates “eta” =0+ Standard Deviation \* N(0.1)
* Price indicates the estimated price in 10 days : =1.942\*exp(log return)

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| Unhedged Simulations | | | | Hedged Simulations | | | |
| # | N(0.1) | Log Return | Price |  | # | N(0.1) | Price |
| 1 | -0.207 | -0.010 | 1.922 | 1 | 1.388 | 0.031 | 2.003 |
| 2 | 0.077 | 0.004 | 1.949 | 2 | 0.153 | 0.003 | 1.949 |
| 3 | 0.663 | 0.033 | 2.006 | 3 | 0.568 | 0.013 | 1.967 |
| 4 | 1.088 | 0.053 | 2.049 | 4 | 0.530 | 0.012 | 1.965 |
| 5 | 0.826 | 0.041 | 2.022 | 5 | 0.275 | 0.006 | 1.954 |
| 6 | -0.351 | -0.017 | 1.909 | 6 | -1.044 | -0.023 | 1.897 |
| 7 | 0.534 | 0.026 | 1.994 | 7 | 0.581 | 0.013 | 1.967 |
| 8 | -0.632 | -0.031 | 1.883 | 8 | 0.298 | 0.007 | 1.955 |
| 9 | 0.721 | 0.035 | 2.012 | 9 | 0.857 | 0.019 | 1.979 |
| 10 | -0.420 | -0.021 | 1.902 | 10 | 1.864 | 0.041 | 2.024 |
| 11 | 0.739 | 0.036 | 2.014 | 11 | -0.731 | -0.016 | 1.911 |
| 12 | -0.129 | -0.006 | 1.930 | 12 | 0.429 | 0.010 | 1.961 |
| 13 | -2.092 | -0.103 | 1.752 | 13 | -0.252 | -0.006 | 1.931 |
| 14 | -0.298 | -0.015 | 1.914 | 14 | -2.700 | -0.060 | 1.829 |
| 15 | 0.902 | 0.044 | 2.030 | 15 | 0.674 | 0.015 | 1.971 |
| 16 | -1.462 | -0.072 | 1.807 | 16 | 1.373 | 0.031 | 2.002 |
| 17 | -0.479 | -0.024 | 1.897 | 17 | -0.735 | -0.016 | 1.911 |
| 18 | 0.980 | 0.048 | 2.038 | 18 | 0.168 | 0.004 | 1.949 |
| 19 | -0.917 | -0.045 | 1.856 | 19 | -0.309 | -0.007 | 1.929 |
| 20 | 0.534 | 0.026 | 1.994 | 20 | 1.917 | 0.043 | 2.027 |
| 21 | 0.068 | 0.003 | 1.948 | 21 | -0.327 | -0.007 | 1.928 |
| 22 | -0.290 | -0.014 | 1.915 | 22 | -1.224 | -0.027 | 1.890 |
| 23 | 0.770 | 0.038 | 2.017 | 23 | 1.352 | 0.030 | 2.001 |
| 24 | -0.142 | -0.007 | 1.928 | 24 | -0.114 | -0.003 | 1.937 |
| 25 | 0.558 | 0.027 | 1.996 | 25 | -0.869 | -0.019 | 1.905 |
| 26 | 2.856 | 0.140 | 2.234 | 26 | -0.602 | -0.013 | 1.916 |
| 27 | -1.089 | -0.053 | 1.841 | 27 | 0.508 | 0.011 | 1.964 |
| 28 | -1.316 | -0.065 | 1.820 | 28 | -1.485 | -0.033 | 1.879 |
| 29 | 0.501 | 0.025 | 1.990 | 29 | -0.653 | -0.015 | 1.914 |
| 30 | 1.424 | 0.070 | 2.083 | 30 | 0.185 | 0.004 | 1.950 |
| 31 | -0.204 | -0.010 | 1.923 | 31 | -1.452 | -0.032 | 1.880 |
| 32 | 0.993 | 0.049 | 2.039 | 32 | 0.115 | 0.003 | 1.947 |
| 33 | -0.593 | -0.029 | 1.886 | 33 | 1.259 | 0.028 | 1.997 |
| 34 | 2.079 | 0.102 | 2.151 | 34 | 0.450 | 0.010 | 1.962 |
| 35 | -0.405 | -0.020 | 1.904 | 35 | 0.469 | 0.010 | 1.962 |
| 36 | -0.676 | -0.033 | 1.879 | 36 | -0.899 | -0.020 | 1.904 |
| 37 | -0.833 | -0.041 | 1.864 | 37 | 0.107 | 0.002 | 1.947 |
| 38 | 0.081 | 0.004 | 1.950 | 38 | -0.737 | -0.016 | 1.910 |
| 39 | 0.508 | 0.025 | 1.991 | 39 | -0.321 | -0.007 | 1.928 |
| 40 | 0.128 | 0.006 | 1.954 | 40 | -1.899 | -0.042 | 1.862 |
| 41 | 1.388 | 0.068 | 2.079 | 41 | -1.008 | -0.022 | 1.899 |
| 42 | -1.044 | -0.051 | 1.845 | 42 | -1.093 | -0.024 | 1.895 |
| 43 | -1.638 | -0.080 | 1.792 | 43 | 0.025 | 0.001 | 1.943 |
| 44 | 1.318 | 0.065 | 2.072 | 44 | -0.322 | -0.007 | 1.928 |
| 45 | -1.036 | -0.051 | 1.846 | 45 | -1.177 | -0.026 | 1.892 |
| 46 | -2.346 | -0.115 | 1.731 | 46 | 1.066 | 0.024 | 1.989 |
| 47 | 3.300 | 0.162 | 2.284 | 47 | 0.434 | 0.010 | 1.961 |
| 48 | 0.782 | 0.038 | 2.018 | 48 | -0.534 | -0.012 | 1.919 |
| 49 | -0.917 | -0.045 | 1.856 | 49 | 0.918 | 0.020 | 1.982 |
| 50 | 0.391 | 0.019 | 1.980 | 50 | 0.143 | 0.003 | 1.948 |
| 51 | -0.789 | -0.039 | 1.868 | 51 | 0.224 | 0.005 | 1.952 |
| 52 | -0.275 | -0.014 | 1.916 | 52 | -0.727 | -0.016 | 1.911 |
| 53 | -1.702 | -0.084 | 1.786 | 53 | -0.311 | -0.007 | 1.929 |
| 54 | 1.749 | 0.086 | 2.116 | 54 | 0.354 | 0.008 | 1.957 |
| 55 | -0.126 | -0.006 | 1.930 | 55 | -0.284 | -0.006 | 1.930 |
| 56 | 0.978 | 0.048 | 2.038 | 56 | -0.604 | -0.013 | 1.916 |
| 57 | -0.496 | -0.024 | 1.895 | 57 | -0.061 | -0.001 | 1.939 |
| 58 | 1.427 | 0.070 | 2.083 | 58 | -0.410 | -0.009 | 1.924 |
| 59 | -2.183 | -0.107 | 1.744 | 59 | -0.093 | -0.002 | 1.938 |
| 60 | -2.390 | -0.117 | 1.727 | 60 | 0.520 | 0.012 | 1.965 |
| 61 | -0.052 | -0.003 | 1.937 | 61 | 0.902 | 0.020 | 1.981 |
| 62 | 1.583 | 0.078 | 2.099 | 62 | -0.498 | -0.011 | 1.921 |
| 63 | -2.105 | -0.103 | 1.751 | 63 | 0.949 | 0.021 | 1.983 |
| 64 | 0.949 | 0.047 | 2.035 | 64 | -1.342 | -0.030 | 1.885 |
| 65 | -2.117 | -0.104 | 1.750 | 65 | 1.834 | 0.041 | 2.023 |
| 66 | 0.982 | 0.048 | 2.038 | 66 | -0.012 | 0.000 | 1.941 |
| 67 | 0.289 | 0.014 | 1.970 | 67 | 0.534 | 0.012 | 1.965 |
| 68 | 2.185 | 0.107 | 2.162 | 68 | -0.944 | -0.021 | 1.902 |
| 69 | -1.745 | -0.086 | 1.782 | 69 | 2.244 | 0.050 | 2.041 |
| 70 | 0.160 | 0.008 | 1.957 | 70 | 0.625 | 0.014 | 1.969 |
| 71 | 0.382 | 0.019 | 1.979 | 71 | 1.045 | 0.023 | 1.988 |
| 72 | -0.182 | -0.009 | 1.925 | 72 | 0.705 | 0.016 | 1.973 |
| 73 | -1.299 | -0.064 | 1.822 | 73 | 0.724 | 0.016 | 1.974 |
| 74 | 1.835 | 0.090 | 2.125 | 74 | -0.457 | -0.010 | 1.922 |
| 75 | -0.334 | -0.016 | 1.910 | 75 | -0.504 | -0.011 | 1.920 |
| 76 | 1.672 | 0.082 | 2.108 | 76 | 0.482 | 0.011 | 1.963 |
| 77 | 0.490 | 0.024 | 1.989 | 77 | -0.113 | -0.003 | 1.937 |
| 78 | -1.005 | -0.049 | 1.848 | 78 | -4.125 | -0.092 | 1.772 |
| 79 | -1.319 | -0.065 | 1.820 | 79 | 0.349 | 0.008 | 1.957 |
| 80 | 0.915 | 0.045 | 2.031 | 80 | 1.124 | 0.025 | 1.991 |
| 81 | 2.046 | 0.101 | 2.147 | 81 | -1.354 | -0.030 | 1.884 |
| 82 | 0.797 | 0.039 | 2.020 | 82 | -0.858 | -0.019 | 1.905 |
| 83 | -0.849 | -0.042 | 1.863 | 83 | 0.467 | 0.010 | 1.962 |
| 84 | 1.843 | 0.091 | 2.126 | 84 | 0.660 | 0.015 | 1.971 |
| 85 | -1.952 | -0.096 | 1.764 | 85 | -0.282 | -0.006 | 1.930 |
| 86 | 0.793 | 0.039 | 2.019 | 86 | 1.293 | 0.029 | 1.999 |
| 87 | 0.140 | 0.007 | 1.955 | 87 | -3.017 | -0.067 | 1.816 |
| 88 | 0.017 | 0.001 | 1.944 | 88 | 1.201 | 0.027 | 1.995 |
| 89 | 0.621 | 0.030 | 2.002 | 89 | -0.407 | -0.009 | 1.925 |
| 90 | -1.921 | -0.094 | 1.767 | 90 | 0.720 | 0.016 | 1.973 |
| 91 | -0.370 | -0.018 | 1.907 | 91 | -1.513 | -0.034 | 1.878 |
| 92 | -0.515 | -0.025 | 1.893 | 92 | 1.425 | 0.032 | 2.005 |
| 93 | 0.399 | 0.020 | 1.980 | 93 | 1.426 | 0.032 | 2.005 |
| 94 | -0.165 | -0.008 | 1.926 | 94 | 1.050 | 0.023 | 1.988 |
| 95 | -0.664 | -0.033 | 1.880 | 95 | 1.518 | 0.034 | 2.009 |
| 96 | 1.130 | 0.055 | 2.053 | 96 | -0.309 | -0.007 | 1.929 |
| 97 | -1.335 | -0.066 | 1.819 | 97 | -1.405 | -0.031 | 1.882 |
| 98 | -0.617 | -0.030 | 1.884 | 98 | 0.225 | 0.005 | 1.952 |
| 99 | 1.071 | 0.053 | 2.047 | 99 | 0.096 | 0.002 | 1.946 |
| 100 | 0.913 | 0.045 | 2.031 | 100 | -1.858 | -0.041 | 1.863 |